Sensitivity analysis through adjoint method: application to the GLORYS reanalysis

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Introduction

In order to improve the Mercator Assimilation System, SAM2, different approaches are developed to analyse the system performance. Sensitivity analysis is one of them. It should help to point out the important regions and variables to control to reduce the forecast error.

The forecast error is measured with the observation misfit. Its sensitivity to the initial conditions of the ocean state and the forcing field is computed with the adjoint model. This will also give us an indication on how the initial conditions are important to optimize compared to forcing fields.

This analysis is applied to the Glorys reanalysis. Glorys is based on the global configuration ORCA1/4°, the model version was NEMO1.9. The next simulation will be based on the NEMO3.0 version. The first reanalysis stream covers the period from 2002 to 2008. Along track SLA (AVISO/SALTO), in-situ temperature and salinity profiles (CORA02) and SST observations are assimilated. The ocean forcing field computation is based on bulk formulae, the atmospheric fields are coming from the ECMWF operational model.

Methodology

The sensitivity analysis is limited to 7 days, the assimilation window chosen for SAM2/Glorys. The gradient of the observation misfit error gives us the sensitivity to the chosen variables, here the initial conditions and forcing fields.

We choose a measure of the error using the square model-observation misfit weighted by the observation error which includes the representativity error. The cost function J is then defined as:

$$J = \frac{1}{2}\sum_{n=1,nTobs} \frac{(T(x,y,z,t) - T^{obs})^2}{\sigma^2_{Tobs}} + \frac{1}{2}\sum_{n=1,nSobs} \frac{(S(x,y,z,t) - S^{obs})^2}{\sigma^2_{Sobs}}$$
$$+ \frac{1}{2}\sum_{n=1,nTobs} \frac{(SLA(x,y,t) - SLA^{obs})^2}{\sigma^2_{SLAobs}} + \frac{1}{2}\sum_{n=1,nSSTobs} \frac{(SST(x,y,t=7days) - SST^{obs})^2}{\sigma^2_{SSTobs}}$$

J has not unit. The sensitivity analysis will tell us how the functional J is changing for **any small perturbation** of the initial conditions (δT, δS, δu, δv, δssh) t=0 or input ocean forcing (δQsr, δQns, δEmP), **in a given background situation**. This will indicate us where a specific effort on reducing the analysis error or forcing error is important.

$$dJ = \frac{\partial J}{\partial T_0}\delta T_0 + \frac{\partial J}{\partial S_0}\delta S_0 + \frac{\partial J}{\partial u_0}\delta u_0 + \frac{\partial J}{\partial v_0}\delta v_0 + \frac{\partial J}{\partial ssh_0}\delta ssh_0$$
$$+ \frac{\partial J}{\partial Qsr}\delta Qsr + \frac{\partial J}{\partial Qns}\delta Qns + \frac{\partial J}{\partial EmP}\delta EmP$$

If we consider the SST misfit, the sensitivity analysis will show to which variable, at which depth and to which forcing variable the SST misfit is the most sensitive. For example, in regions of strong currents, the SST misfit will be sensitive to temperature changes in the "upward" region; in different region it can be more sensitive to deeper temperature than to the heat flux.

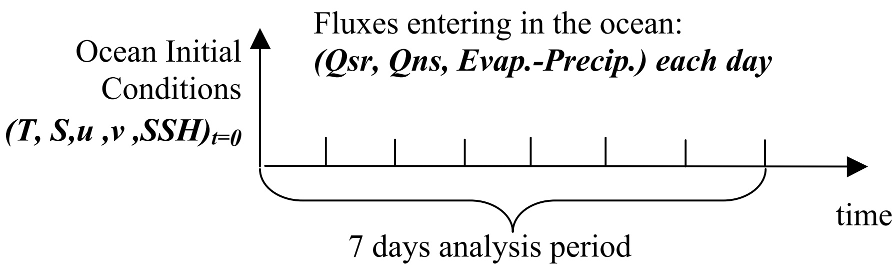
The impact on J of a change of 1°C in one grid point will depend on the number of the observations "sensitive" to this grid point. This is especially true for the in situ observations. For SST observations and SLA observations, the geographical repartition is more homogeneous than for the in situ observations.

The adjoint model provides a unique tool to compute the derivative of a function to the input variables. Here we will consider the initial conditions and daily atmospheric forcing fields. The observation misfit is "propagated" backward in time from the observation instant to the initial state by the adjoint model. At the initial time, the gradient of the cost function to model state variable is given by the value of the adjoint variable:

$$\frac{\partial J}{\partial T_0} = T_0^{ad}, \frac{\partial J}{\partial S_0} = S_0^{ad}, \frac{\partial J}{\partial u_0} = u_0^{ad}, \frac{\partial J}{\partial v_0} = v_0^{ad}, \frac{\partial J}{\partial ssh_0} = ssh_0^{ad},$$

$$\frac{\partial J}{\partial Qsr_d} = \overline{Qsr_d}^{ad}, \frac{\partial J}{\partial Qns_d} = \overline{Qns_d}^{ad}, \frac{\partial J}{\partial EmP_d} = \overline{EmP_d}^{ad}$$

The fluxes entering in the ocean, computed from atmospheric fields and bulk formulae, are added to the control vector. We compute the sensitivity to the daily fluxes. The adjoint flux variables are sum up over each day.



To assess the sensitivity of the observation misfit in the Glorys system, we need to use the observations assimilated with their prescribed errors and not a different data set with different selection criteria.

The model equivalents to the observations are computed during the integration of the NEMO ocean model. The information useful for the analysis is stored in memory or in files, especially if the model integration and the analysis are done by different executables. In the NEMOVAR system, the observation information is stored in "feedback files", in the SAM2 system for Glorys they are exchanged in memory. For diagnostic purposes, observation information is written in "ola" files at the end of the SAM2 analysis. Then, we need to read the useful information on assimilated observations in the ola files and write feedback files. Even if the same kind of information is stored in both files their format greatly differs.

The location, time and type of observation are stored as well as the observation value and the model equivalent. The standard deviation of the observation error is needed for the sensitivity analysis. The standard way in NEMOVAR is to either set it to a contant value in the namelist or to read it from a gridded file. As it is already stored in the ola files, we directly transferred this information into the feedback files, thanks to the possibility offered by the data feedback structure to store additional and external relevant information in additional array.

The observation operators were also chosen to be equivalent to the one used in the Glorys reanalysis to compute the observation misfit.

Experimental setup

One of the test cases for the NEMOVAR variational assimilation code is based on the ORCA2° configuration. We adapted it to the ORCA 1/4° configuration. Many numerical schemes are different in the two configurations. Few of the adjoint routines necessary for ORCA ¼° did not exist in the previous version. They were developed so we were able to run the adjoint model with the proper options. Those options are set in the namelist or via the CCP keys. We first done sensitivity experiment with the ORCA 2° configuration using the test case provided with the NEMOVAR code. We move afterward to the ORCA ¼° configuration.

The NEMO ocean model is compiled with the GLORYS options and executed with the same namelist and input files, such as bathymetry, coordinates, runoffs, forcing fields and restarts… The "trajectory" of the full non-linear model, NEMO, is regularly stored in order to be used by the adjoint model. Indeed, the adjoint model is build from the tangent-linear model, which is computed by linearization of the full non-linear model around a reference trajectory. As NEMO contains non linear terms, the integration of the adjoint model requires the knowledge of the reference trajectory. Few simplifications where done to build the tangent-linear model, in particular the vertical mixing coefficient and the slope of the isopycnals are supposed not to change compare to the direct model and are stored in the reference trajectory file. The observations values, their model equivalent, quality control information are stored in the feedback files to be used during the adjoint model integration.

Even if we have the restart, namelist and input files of the Glorys1v1 simulation, it is not possible to do the exact same experiment because an IAU was used to incorporate the increment, even in the forecast phase. We still did the first experiments with Glorys restart; the reference trajectory used to linearize the tangent and adjoint model was then an approximation. We will later use the Glorys daily outputs as reference trajectory.

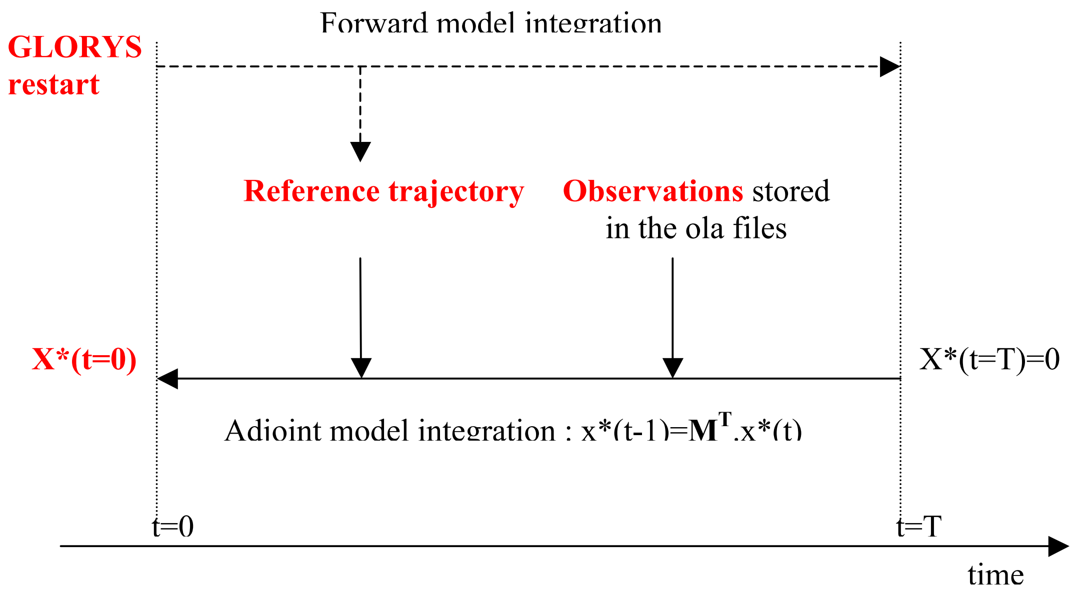


Figure 1: Experimental setup for the GLORYS sensitivity analysis

The code is running on the IBM super computer at ECMWF. The domain is divided in 11x27 processors as for the forced model simulations with ORCA025. The additional memory cost does not increase the need in terms of number of processor.

Results

We present here the results of the sensitivity analysis done for the 8th of March 2006. We did an analysis for each type of observations.

SST misfit sensitivity

We consider the sensitivity to the SST assimilated at the end of the 7 day period.

$$J = \frac{1}{2} \sum_{n=1,nSSTobs} \frac{(SST(x,y,t=7days) - SST^{obs})^2}{\sigma^2_{SSTobs}}$$

The adjoint model is integrated backward to the initial time in order to compute the gradient of J respect to the initial condition (sensitivity to the initial condition). The adjoint linear dynamic shows that the SST is very sensitive to change in temperature close to the surface and even at depth in some regions.

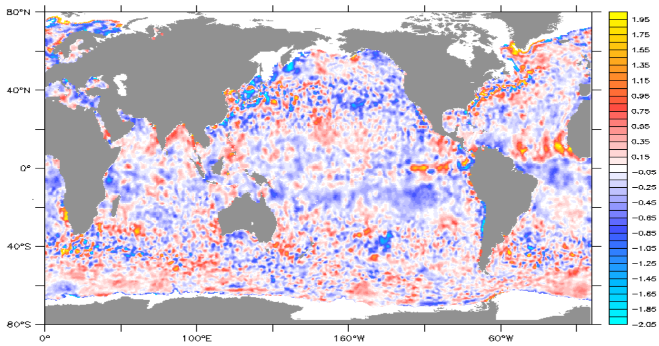


Figure 2: model SST - SST observations @ day=7

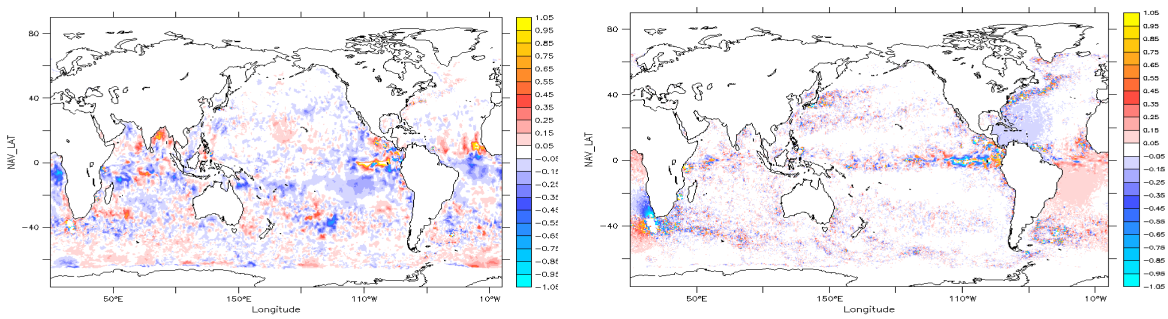


Figure 3: initial temperature and salinity sensitivity for J at the surface weighted by the thickness of the grid cell

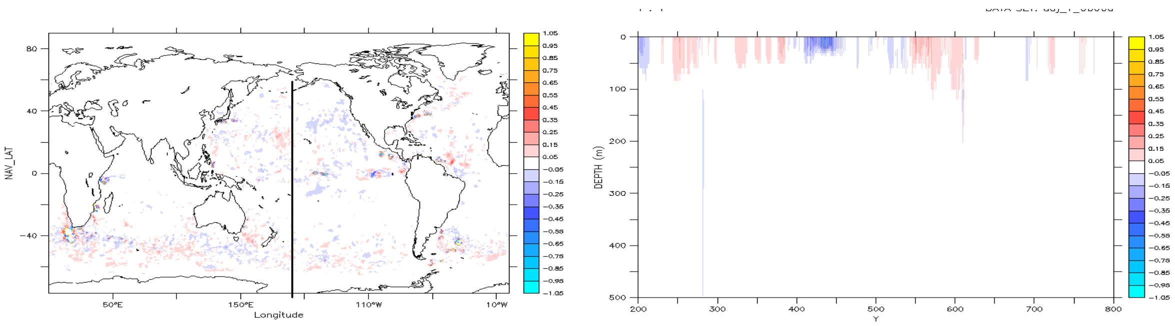
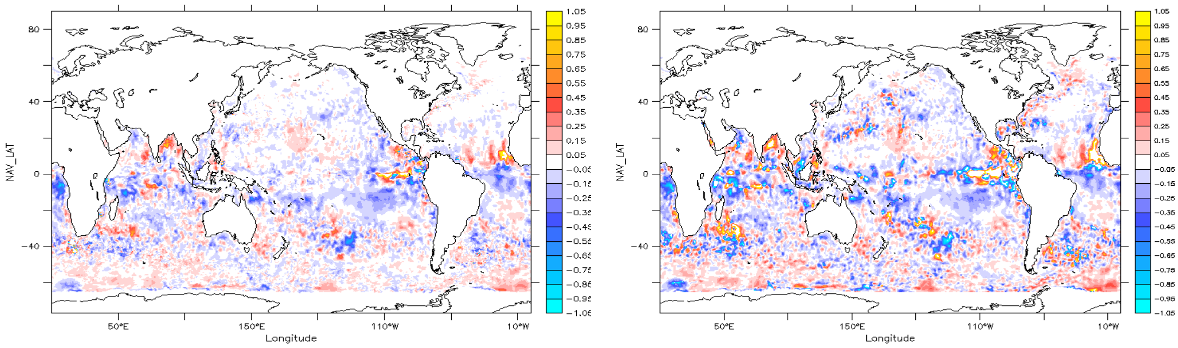


Figure 4: initial temperature sensitivity for J at 150m, weighted by the thickness of the grid level and a vertical section at 175°W



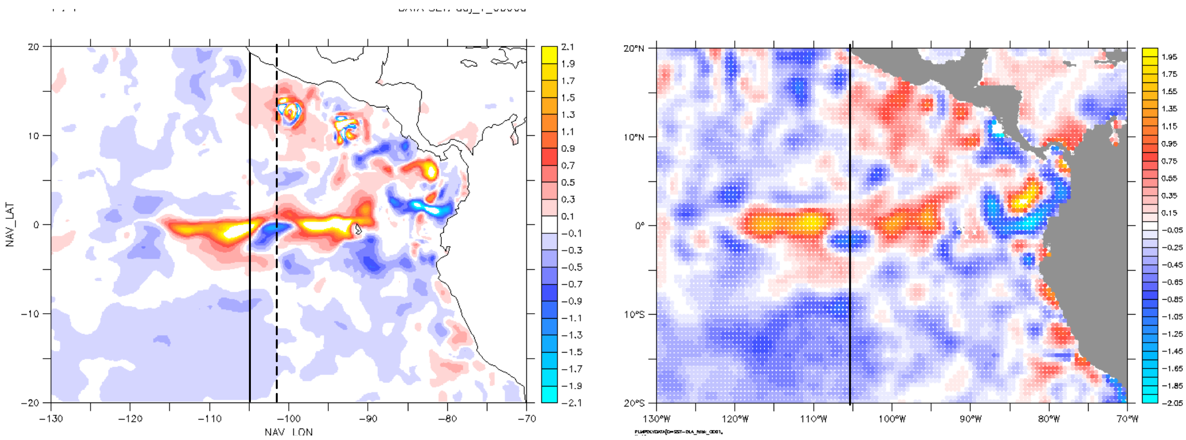
dJ/dQsr (day = 1) dJ/dQsr (day = 7)

Figure 5: solar radiative heat flux sensitivity for the first and last days

The same approach is used to compute the gradient of *J* respect to the heat fluxes. The sensitivity to the heat fluxes of the first day is comparable to the sensitivity to the initial surface temperature. It is mainly concentrated in the southern hemisphere, in the circumpolar region and equatorial belt. The sensitivity to the heat fluxes of the last day, the day of the SST observations, is higher than those of the first day but the patterns are similar.

The sensitivity of the SST misfit to the heat flux is higher in regions where the mixing is weak and the MLD shallow: the impact of the atmospheric heating or cooling is localized at the surface. In the southern hemisphere, the shallower MLD regions correspond to low values of the eddy diffusivity coefficient, themselves linked to "low wind speed" values. In the northern hemisphere, the mixed layer depth is larger than in the southern hemisphere and the vertical eddy diffusivity coefficient is uniformly high. The SST misfit is much less sensitive to the heat flux in the northern hemisphere where the mixed layer depth is larger and associated with stronger vertical eddy diffusivity. The ocean current advection seems to control less the heat propagation compared to the vertical mixing on the 7 day period.

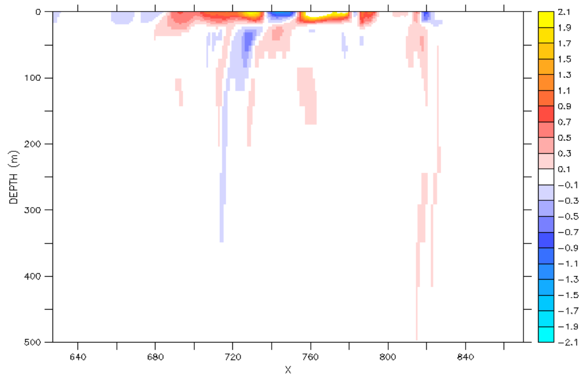
We now focus on a specific region, the western equatorial pacific, where the SST sensitivity is high and the dynamic plays an important role over 7 days at ¼° spatial resolution. The goal is to look at the "covariance" build by the adjoint model during the 7-day backward integration.



$\partial J/\partial T(t{=}0,k{=}1)$ SSTmod-SSTobs (day = 7)

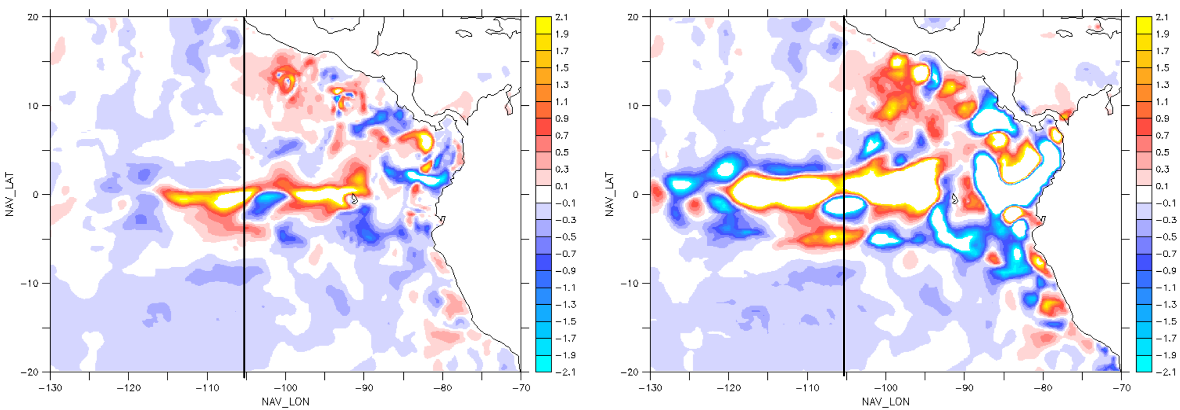
Figure 6: SST sensitivity to surface initial temperature, on the left, SST misfit on the day 7, on the right.

The SST square misfit is more sensitive to the initial temperature along the equator than away from it. We can see the backward advection by the adjoint model. When looking at the vertical structure of the sensitivity, we see that the SST misfit at the day 7 is sensitive not only to the temperature in the mixed layer depth but also to the initial temperature at depth in some place. The origin of those patterns can be interpreted in terms of wave signal.



$\partial J/\partial T(y{=}0°N)$

Figure 7: vertical section of the SST sensitivity to the initial temperature, weighted by the level thickness.



$\partial J/\partial Q_{ns}(day{=}1)$ $\partial J/\partial Q_{ns}(day{=}7)$

Figure 8: SST misfit sensitivity to heat flux of the first and last days

The sensitivity to the heat flux of the day of the observed SST is much higher in the region close to the equator than those of the first day. This is not the case outside the equator on the west.

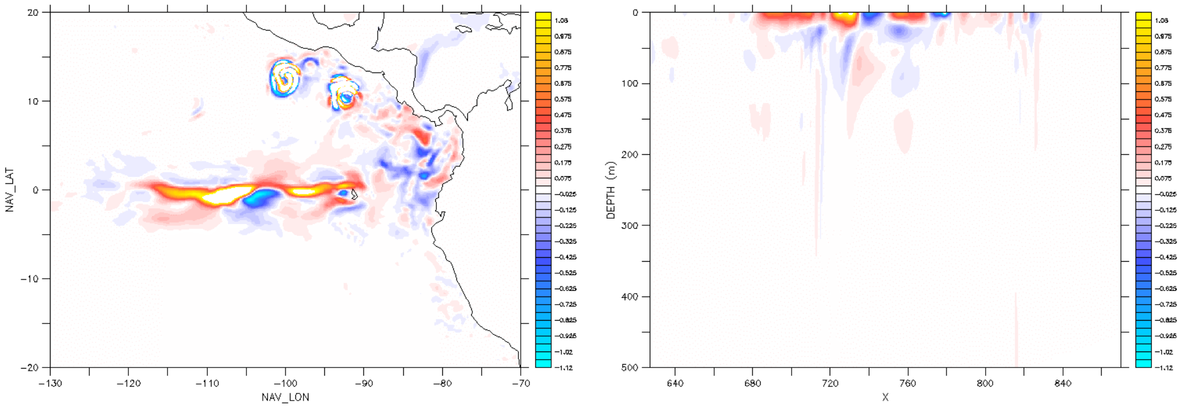


Figure 9: SST sensitivity to the initial zonal velocity

The SST square misfit is sensitive to the zonal velocity initial conditions only in a restricted area along the equator but also north of it, close to the coast. The same vertical patterns as the sensitivity to the temperature at depth are found.

Altimetry misfit sensitivity

We now look at the SLA misfit sensitivity. Along track sea level anomalies are assimilated during the entire time period. The Figure 10 shows the SLA misfits: each point corresponds to an observation location.

$$J = \frac{1}{2} \sum_{n=1,nTobs} \frac{(SSH(x,y,t) - MDT - SLA^{obs})^2}{\sigma^2_{SLAobs}}$$

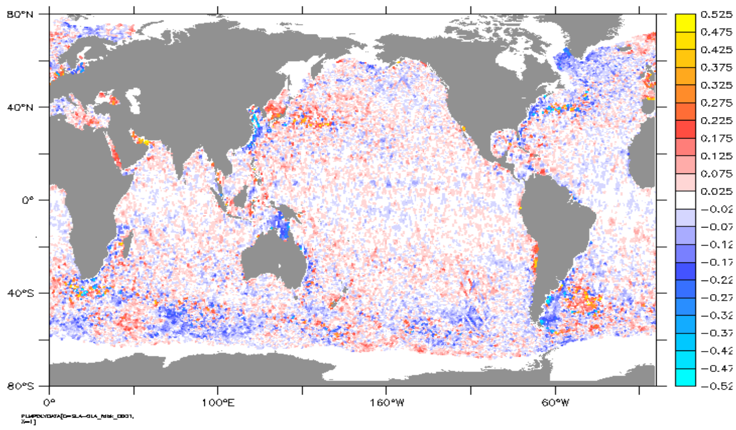


Figure 10: SLA mod - SLA obs.

The SLA misfit is sensitive to the salinity and temperature initial conditions over the entire water column. The partition on the salinity and temperature is driven by their relative contribution in density. Without any normalization, the adjoint values are larger when going deeper in the ocean. This is not the case after applying a weighting depending at least on the vertical thickness of the grid cells. It shows how important is the choice of a norm.

The misfit is mostly insensitive to the initial SSH conditions: only the global mean of the SSH is important. Local changes in the SSH are quasi instantaneously transformed in density and velocity changes.

Over a 7-day period, the SLA misfit is not very sensitive to the atmospheric fluxes which influence only the surface layer at this time scale.

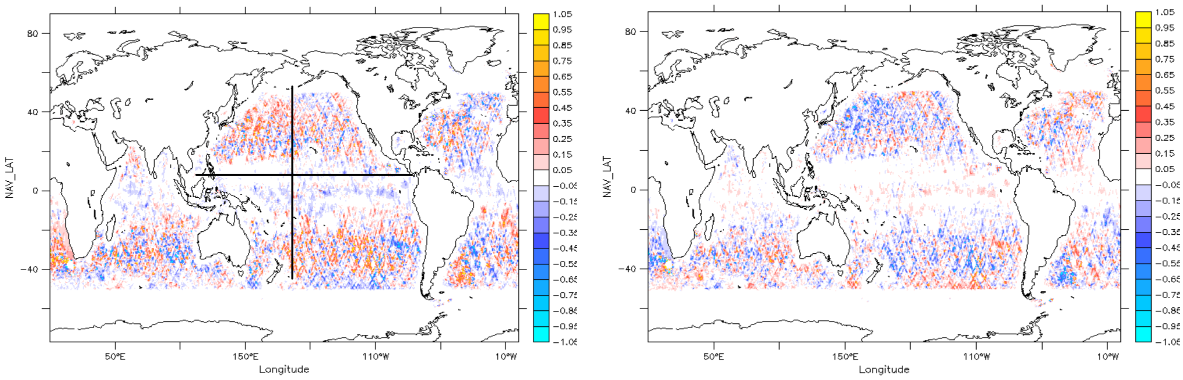


Figure 12: ∂J(SLA)/∂T(0m) and ∂J(SLA)/∂S(0m)

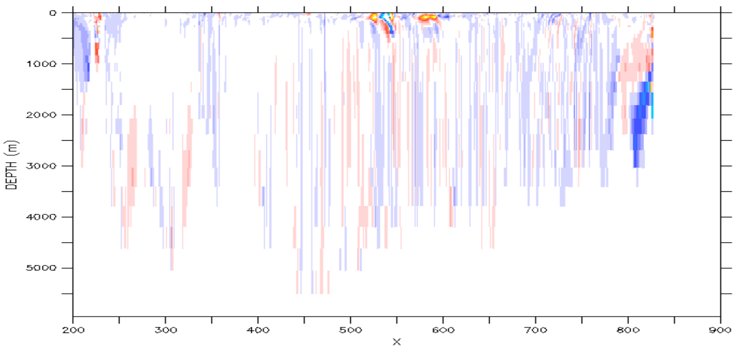


Figure 13: vertical section of ∂J(SLA)/∂T at 175°W

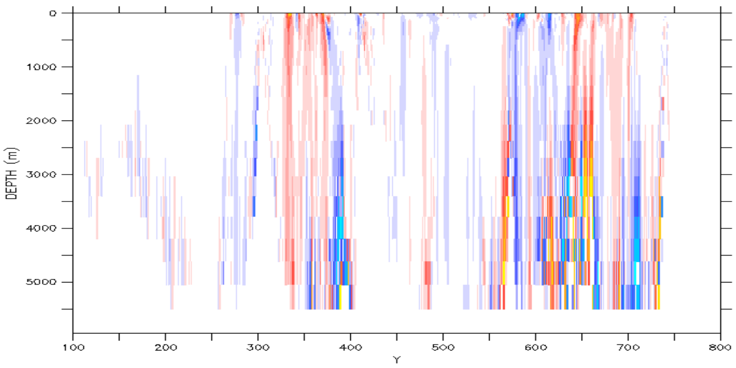


Figure 14: vertical section of ∂J(SLA)/∂T at the equator in the Pacific Ocean

In situ misfit sensitivity

The sensitivity to the in situ observations, from the CORA database, was also computed.

$$J = \frac{1}{2}\sum_{n=1,nTobs}\frac{(T(x,y,z,t)-T^{obs})^2}{\sigma^2_{Tobs}} + \frac{1}{2}\sum_{n=1,nSobs}\frac{(S(x,y,z,t)-S^{obs})^2}{\sigma^2_{Sobs}}$$

The Figure 15 shows the in situ observation sensitivity to the temperature field. The locations of the samples are clearly identifiable. Dirac functions forced the adjoint model at the time and location of the observations. Waves are generated and quickly propagate all over the basin.

Different solutions are proposed:

- Increase the diffusion coefficient in the adjoint as it is done when the assimilation window increase to several month and numerical instabilities develop,
- change the observation operator to add spatial correlation,
- choose to not take into account in situ observations over the first 5 days to let time to the waves to dissipate. They may be still a problem when the forcing fields will be added in the control vector.

In the context of data assimilation, additional information is added through the background term imposing a priori hypothesis on correlation scales and covariance for the control variables.

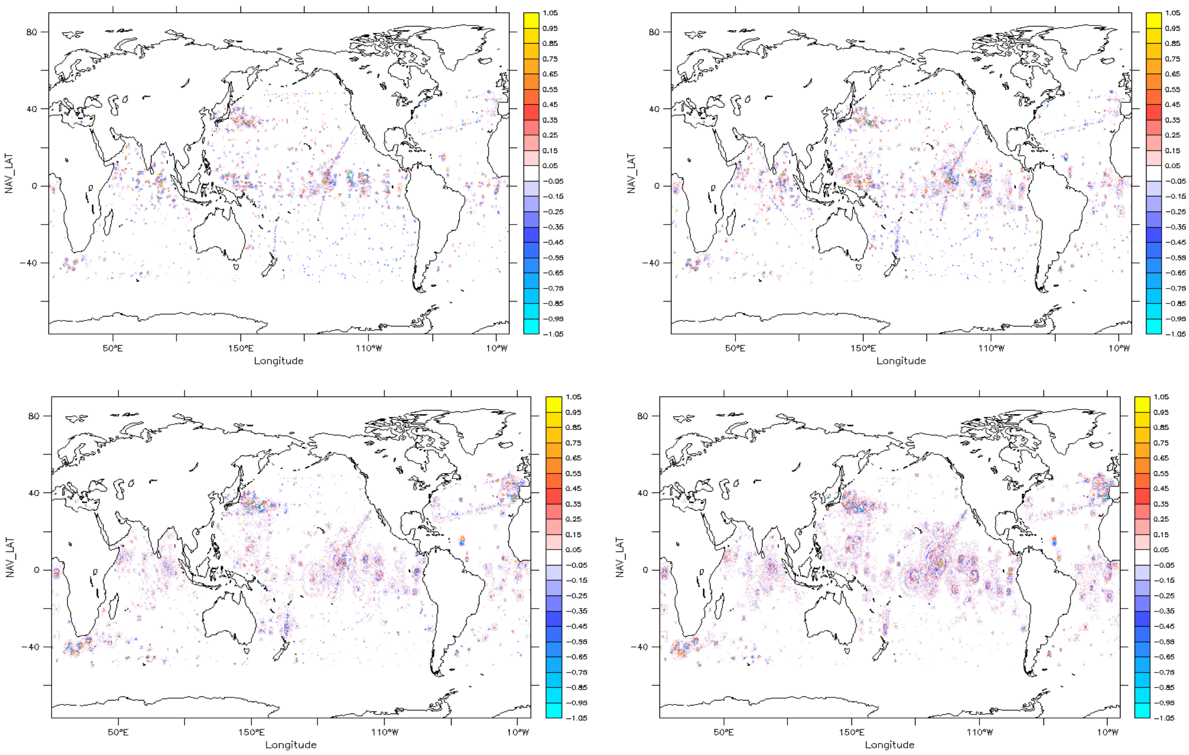


Figure 15: ∂J(in situ)/∂T(t=0) at different depths: 0 m, 50 m, 500 m, 2000 m.

Conclusions

Different remarks can be made when looking at the presented results.

The SST misfit sensitivity computed only at the end of the 7 day period allow us to clearly see the backward propagation of a surface information. The horizontal advection over 7 day in

a ¼° global ocean configuration does not play an important role, except in few regions. The vertical propagation of the misfit can be significant at some locations.
The heat fluxes sensitivity is most important when the Mixed Layer Depth shallow, linked with low vertical mixing coefficient.
The SST misfit is more sensitive to the forcing on the day 7 than the surface temperature initial conditions.

The SLA misfit is sensitive to the initial T and S perturbation over the entire water column. The sensitivity ratio between T and S corresponds to their contribution in the dynamic height. Some high sensitivity appears close to steep bottom bathymetry. The SLA misfit in not sensitive to the SSH initial conditions perturbation, only to their mean.

The in situ sensitivity shows a dynamical response of the adjoint, which is difficult to interpret. It is clear that in an assimilation context, the specificity of the in situ observations, *i.e.* sparse local observations, should be taken into account and a model error covariance carefully defined to extrapolate the information “around” the observation location.